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Tuning Hyperparameters with

Bayesian Optimization



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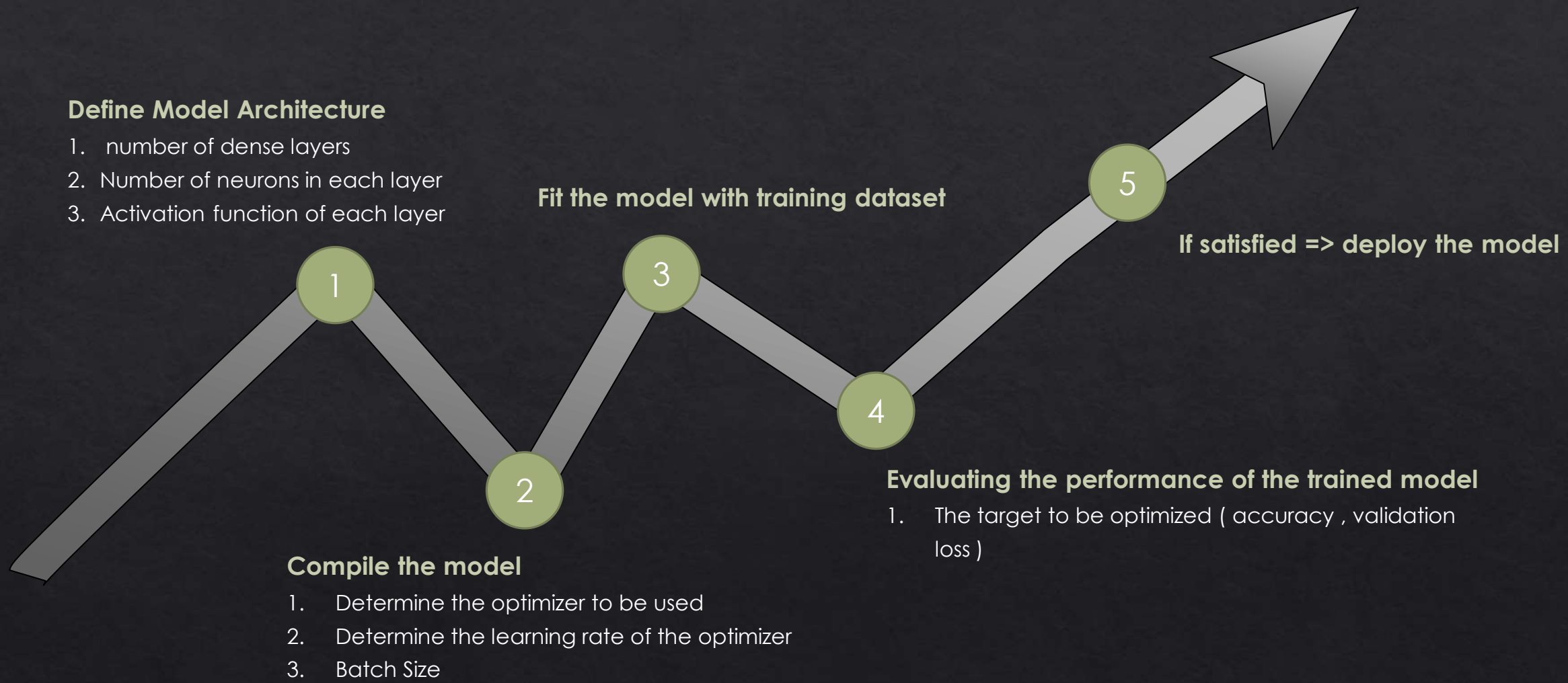


A c t u a l S i t u a t i o n s

part 1

Tuning Hyperparameters with Bayesian Optimization

Actual Situations



Actual Situations

- The training process requires a lot of computing resource.
- It's costly to try different sets of hyperparameters .
 - >> takes iterations of training process shown in the previous figure .
- Bayesian Optimization
 - >> minimize the iterations of training process needed

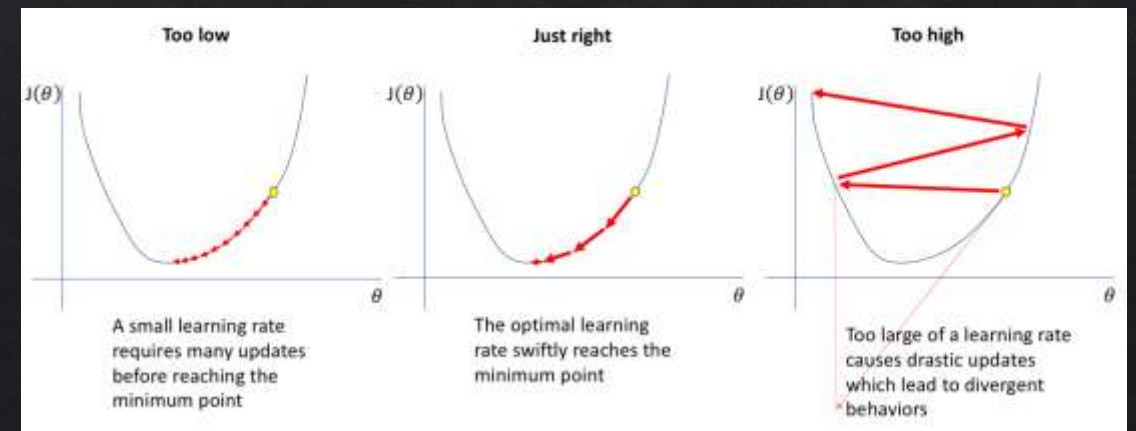
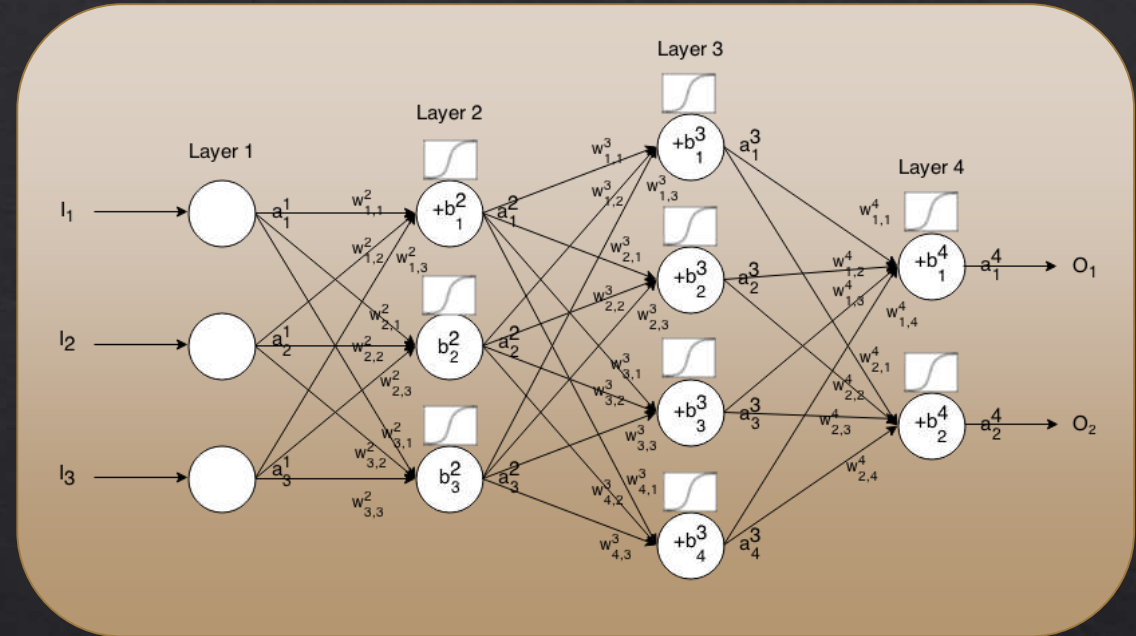
Actual Situations

Hyperparameters (non - trainable):

- Learning Rate
- Activation function of each layer
- Dropout rate of target layer
- Number of neurons in Dense layer

Algorithms :

- Grid Search
- Random Search
- Bayesian Optimization





Motivation & Goal

part 2

Tuning Hyperparameters with Bayesian Optimization

Motivation



Develop complex deep neural networks.

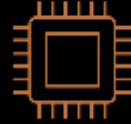


A Probabilistic way of tuning hyperparameters .

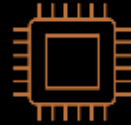


Accelerate the life cycle of producing a model.

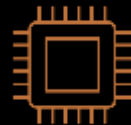
Goal



Get a deeper insight into Bayesian Optimization
(understanding the mathematics behind)



Implementing it through python



Notice its pros and cons and give proper solutions



Structure of B.O.

part 3

Tuning Hyperparameters with Bayesian Optimization

Structure of Bayesian Optimization

Big Picture



- **Bayesian Optimization**

Gaussian Process:

Probabilistic model capable of predicting target function with prior belief .

Acquisition Function:

Strategy to find the next set of hyperparameters worth trying .

Structure of Bayesian Optimization

Gaussian Process

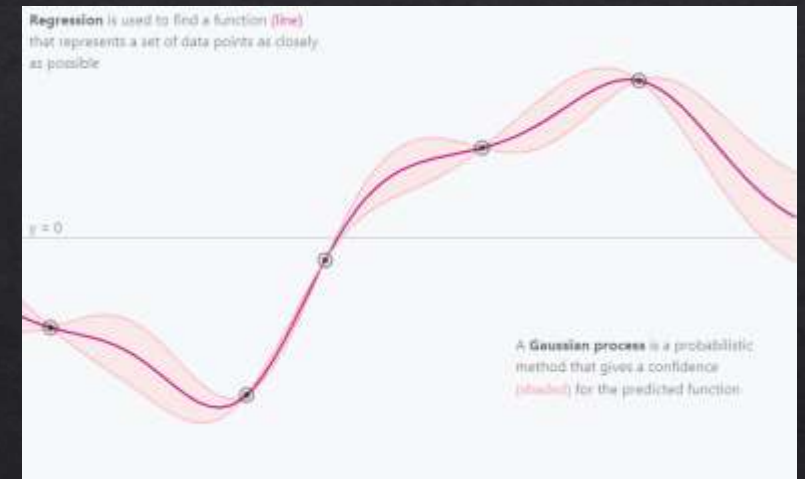
➤ Core Concept

$$\begin{bmatrix} f(x') \\ f(x_1) \\ f(x_2) \\ \vdots \\ f(x_{n-1}) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} k(x', x') & k(x', X)^T \\ k(x', X) & K_{xx} \end{bmatrix} \right) \quad k(x', X) = \begin{bmatrix} k(x', x_1) \\ \vdots \\ k(x', x_n) \end{bmatrix}$$

➤ Determine the kernel function used

➤ Estimating Posterior with Prior Knowledge

- $f(x')|f(X) \sim N(k(x', \cdot)^T K_{xx}^{-1} f(X), k(x', x') + k(x', X)^T K_{xx}^{-1} k(x', X))$
- $\mathbb{E}(f(x')|f(X)) = \sum_{i=1}^n K_{xx}^{-1} f(x_i) k(x', x_i)$



Structure of Bayesian Optimization

Gaussian Process

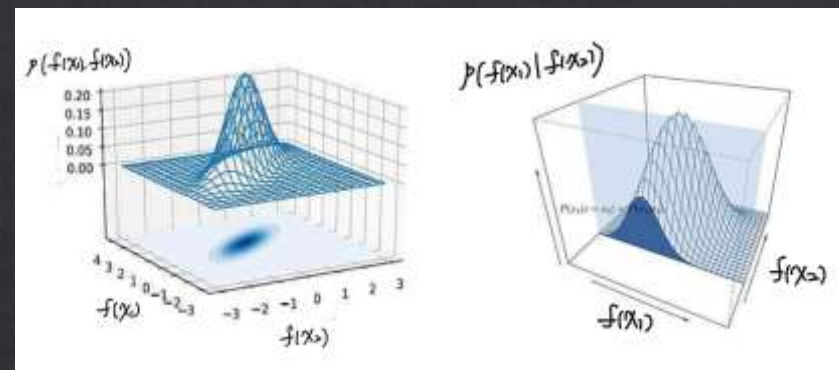
➤ Multivariate Normal Distribution

$$\mathcal{N}(x | \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)\right]$$

➤ Kernel Function

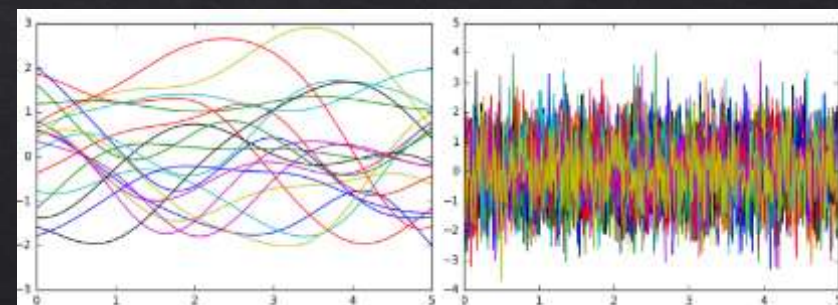
ex: RBF kernel

$$k(x_i, x_j) = \exp\left(\frac{-(x_i - x_j)^2}{2r^2}\right) \quad r: \text{length scale of the kernel}$$



$f(x)$:

performance of the model trained using the specific set of hyperparameters



Structure of Bayesian Optimization

Acquisition Function

➤ **Goal** : determine the next set of hyperparameters used on training model

➤ **Probability of improvement (POI):**

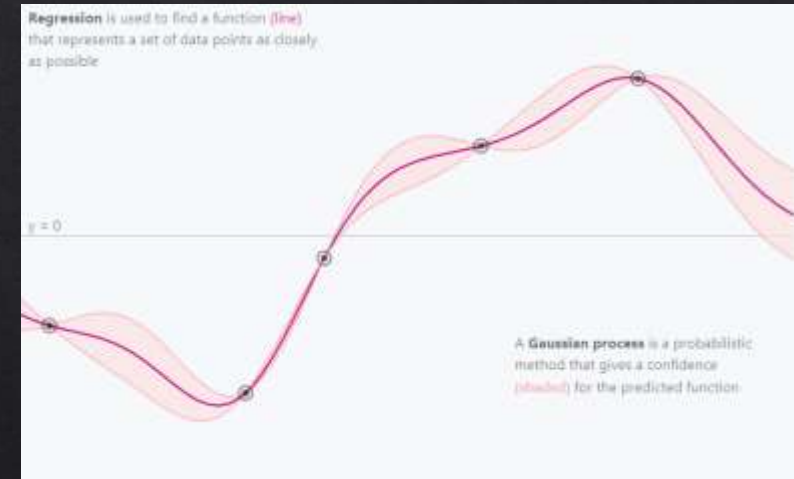
$$POI(x) = P(f(x) \geq f(x^+)) = \phi\left(\frac{\mu(x) - f(x^+)}{\sigma(x)}\right)$$

➤ **Expected improvement (EI) :**

$$EI(x) = \begin{cases} (\mu(x) - f(x^+))\phi(Z) + \sigma(x)\phi(Z) & \text{if } \sigma(x) > 0 \\ 0 & \text{if } \sigma(x) = 0 \end{cases}$$

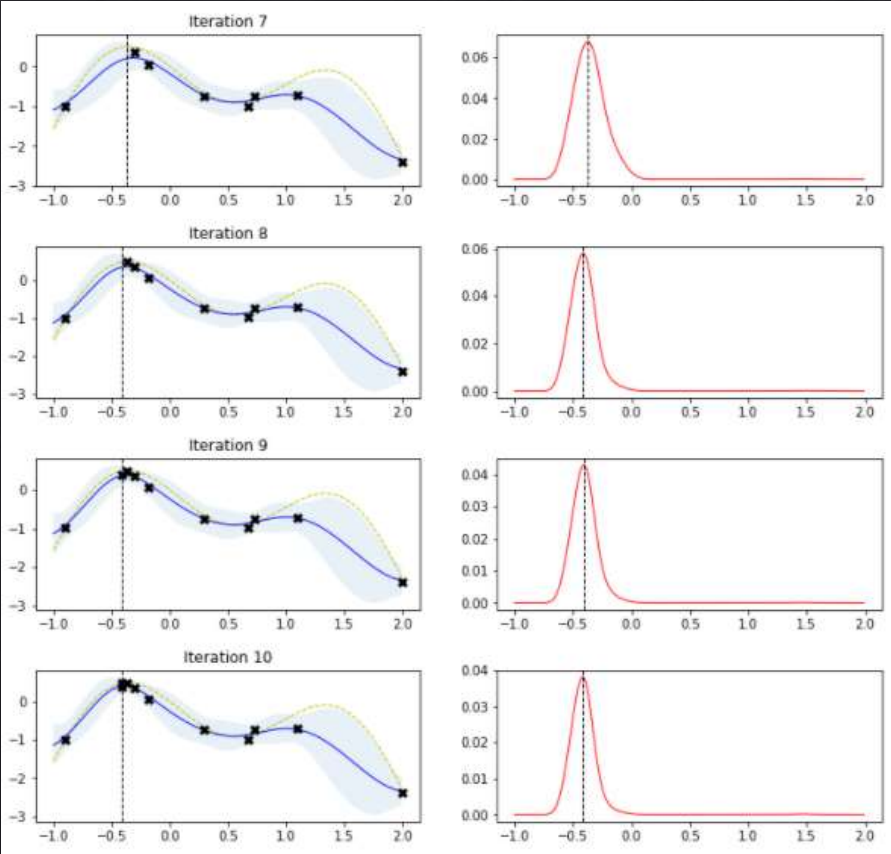
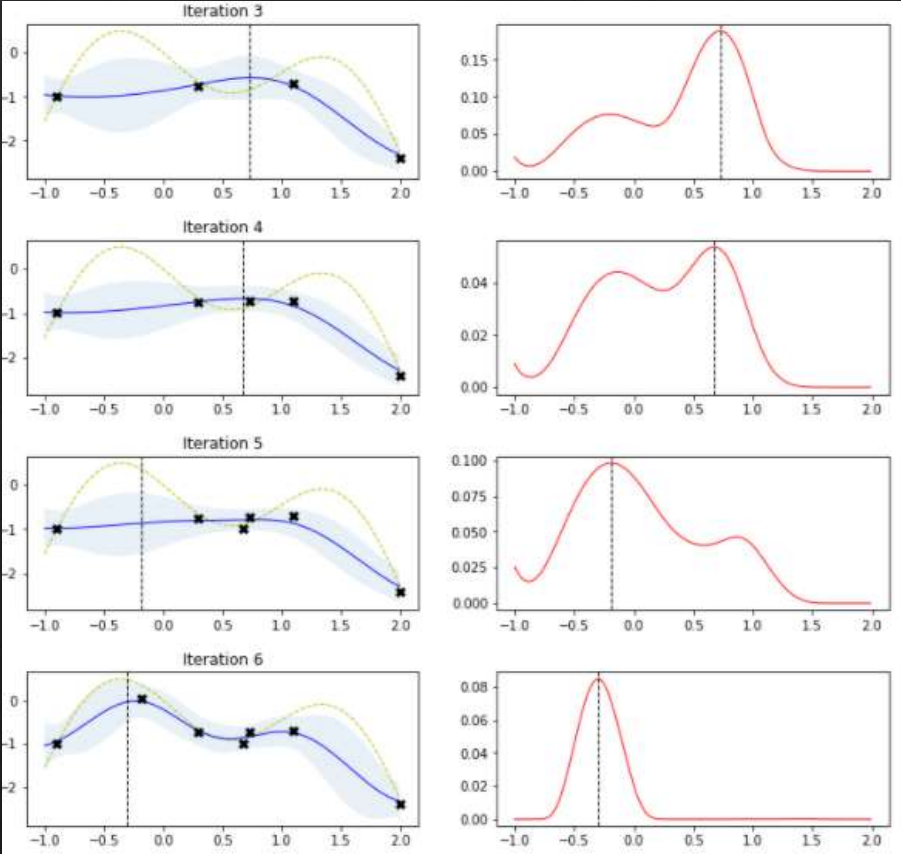
➤ **Upper confidence bound (UCB):**

$$UCB(x) = \mu(x) + \kappa\sigma(x)$$



Structure of Bayesian Optimization

Full View





A p p l i c a t i o n o n M n i s t

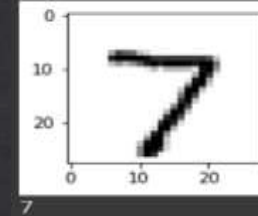
part 4

Tuning Hyperparameters with Bayesian Optimization

Application on Mnist model

- After implementation of Bayesian Optimization through python , I applied it to a Mnist model as my first try .
- Mission for Bayesian Optimization :
Tuning dropout rate and learning rate of the model to optimize the accuracy of the test dataset .

```
1 import numpy as np
2 probability_model(x_test[:5])
3 plot_image(x_test[0].reshape(28, 28))
4 result=probability_model.predict(x_test[0:1])
5 print(np.argmax(result))
```



```
def get_model(input_shape,dropout_rate=None):
    model=tf.keras.models.Sequential()
    model.add(1.Flatten(input_shape=input_shape))
    model.add(1.Dropout(dropout_rate))
    model.add(1.Dense(128,activation='relu'))
    model.add(1.Dense(10,activation='softmax'))
    return model
```

<u>Aa</u> Iteration	# Accuracy of test dataset	# dropout rate	# learning rate
<u>12</u>	0.9762	0.1764	0.002729
<u>13</u>	0.9766	0.1775	0.00219



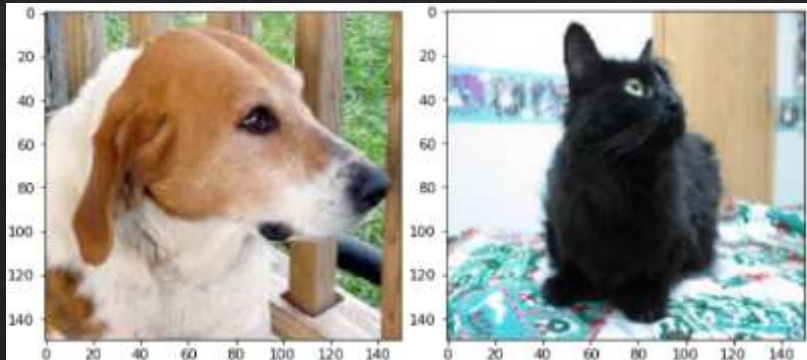
K N N o n c l a s s i f y i n g
d o g s a n d c a t s

p a r t 5

Tuning Hyperparameters with Bayesian Optimization

KNN on classifying dogs & cats

- Mission for Bayesian Optimization :
Determine the number of dense layers used and the dropout rate of each dense layers while learning rate would be chosen as well .
Goal : minimize the validation loss .



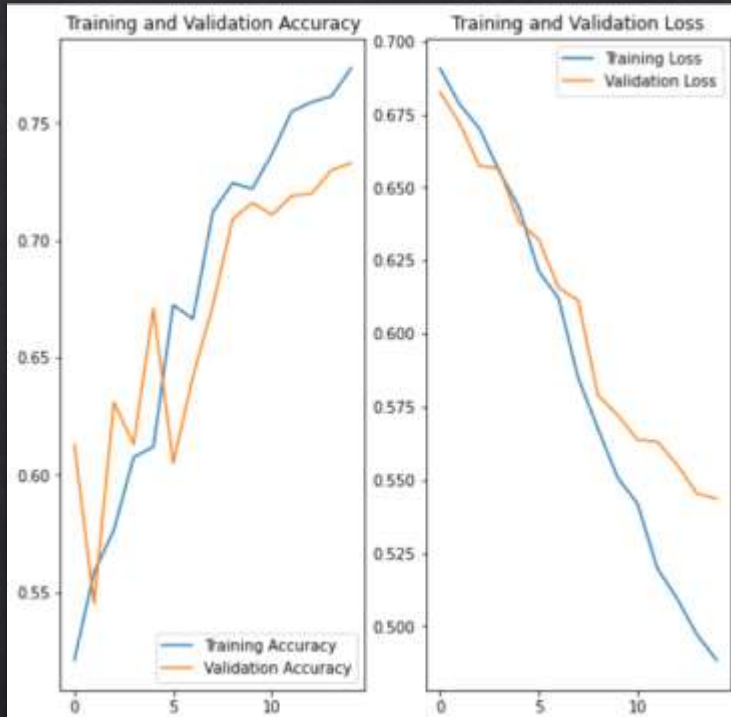
```
def get_model(NUM_DENSE=1,dropout_rate=0):  
    model=tf.keras.models.Sequential()  
    model.add(l.Conv2D(32,(3,3),activation='relu',input_shape=(150,150,3)))  
    model.add(l.MaxPooling2D(2,2))  
  
    model.add(l.Conv2D(64,(3,3),activation='relu'))  
    model.add(l.MaxPooling2D(2,2))  
  
    model.add(l.Conv2D(128,(3,3),activation='relu'))  
    model.add(l.MaxPooling2D(2,2))  
  
    model.add(l.Flatten())  
    for i in range(NUM_DENSE):  
        model.add(l.Dropout(dropout_rate))  
        model.add(l.Dense(512,activation='relu'))  
    model.add(l.Dense(2))  
  
    return model
```

KNN on classifying dogs & cats

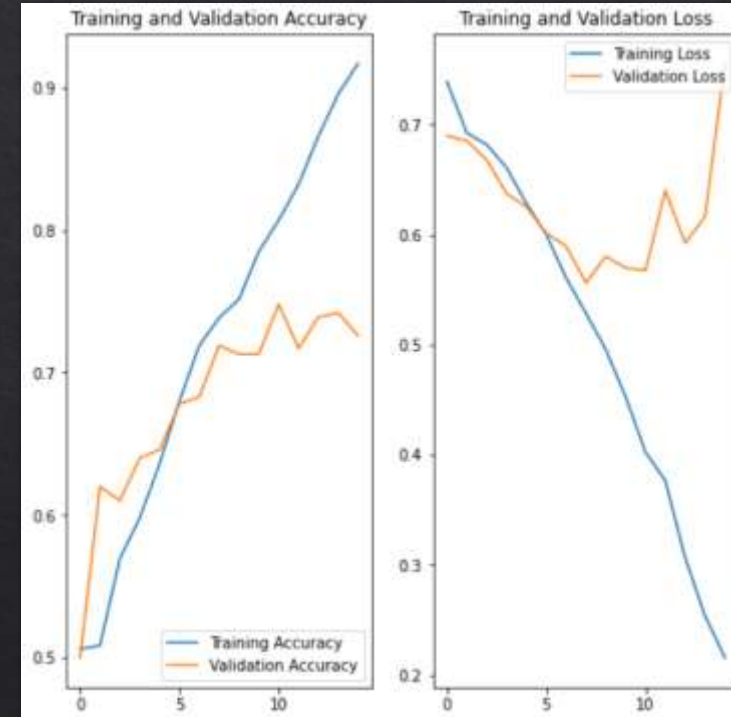
Iteration	# Number of Dense Layers	# Dropout Rate of Dense Layer	# Learning Rate	# Test Set Accuracy	# Test Loss
Iteration 1	3	0.7042271454095106	0.009998867689308286	0.5	0.6931524276733398
Iteration 2	2	0.3027291235719791	0.009085847911788902	0.5	0.6931489109992981
Iteration 3	1	0.44189250893013343	0.006072002005116367	0.5950000286102295	0.6703671813011169
Iteration 4	0	0.8774479954751082	0.0001	0.7329999804496765	0.5435753464698792
Iteration 5	7	0.20441075994440278	0.0001	0.718999981880188	1.0654526948928833
Iteration 6	0	0.20512040660294945	0.0001	0.7179999947547913	0.561133861541748
Iteration 7	0	0.878349648606946	0.0001	0.7009999752044678	0.56093430519104
Iteration 8	0	0.814340423014835	0.0001	0.7070000171661377	0.5644177198410034
Iteration 9	5	0.88642096332855	0.0001	0.7160000205039978	1.0443283319473267
Iteration 10	0	0.5876140250785195	0.0001	0.6930000185966492	0.5836883187294006
Iteration 11	3	0.6649025612669275	0.0001	0.7039999961853027	0.7287527918815613
Iteration 12	6	0.895918584305263	0.0001	0.7149999737739563	0.9349175691604614
Iteration 13	2	0.8922765480338355	0.0001	0.7170000076293945	0.6769360303878784

KNN on classifying dogs & cats

Bayesian version



Model trained by others





P r o b l e m o f l o c a l m a x i m u m

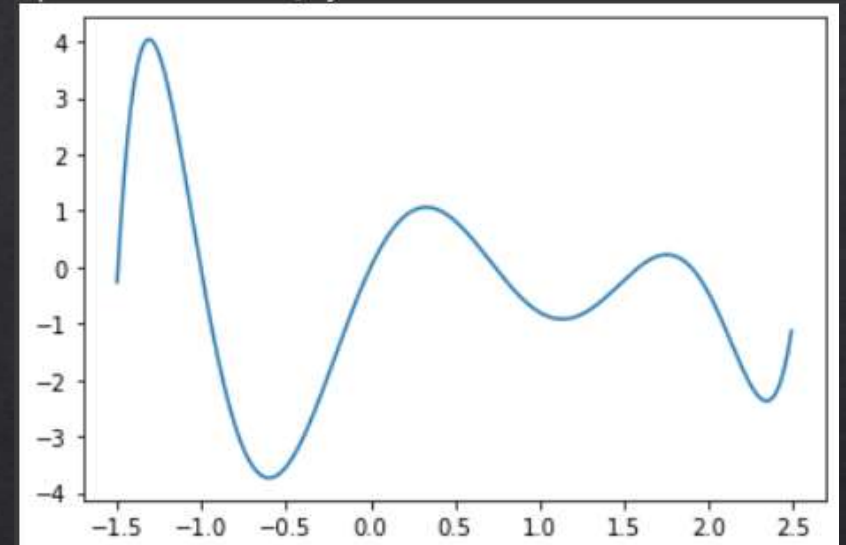
p a r t 6

Tuning Hyperparameters with Bayesian Optimization

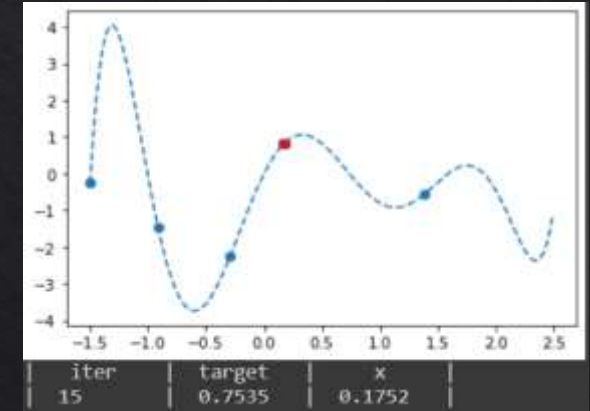
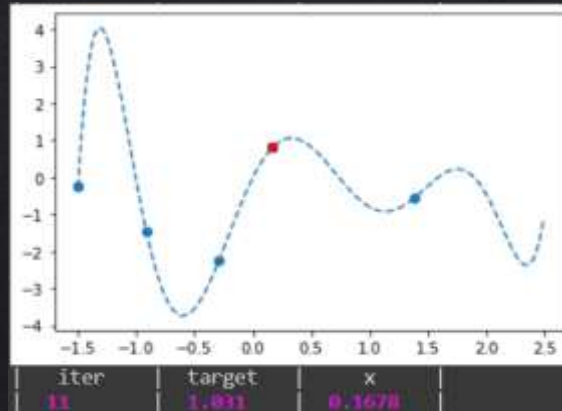
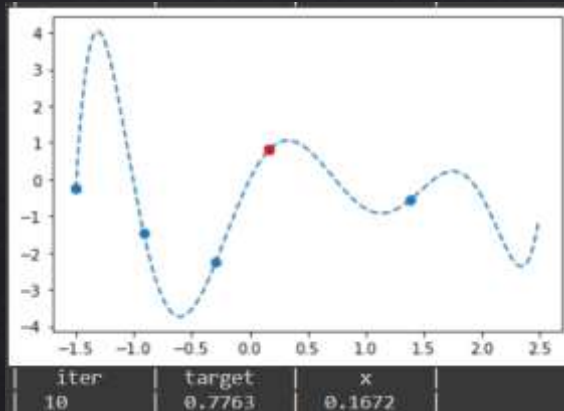
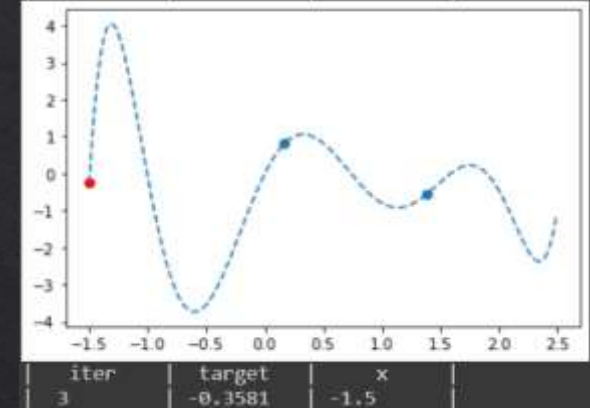
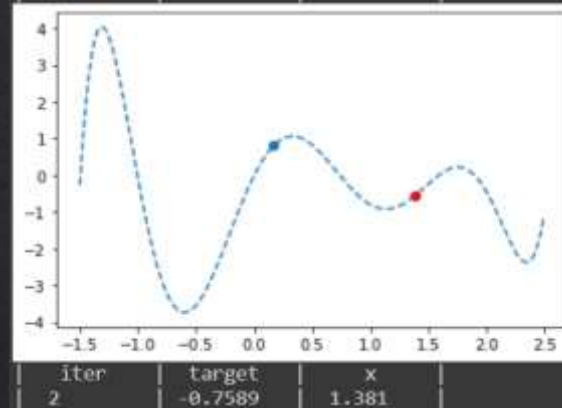
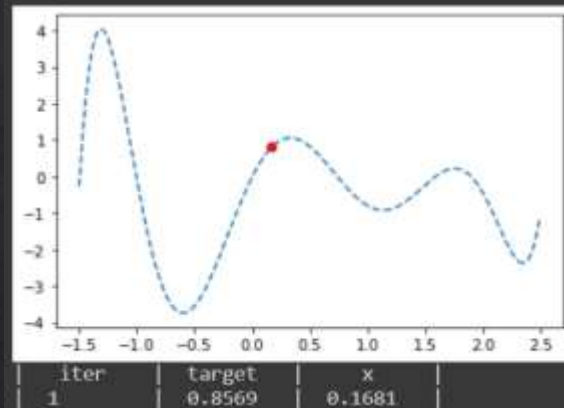
Problem of local maximum

- The target function which Bayesian Optimization aims to find the maximum value of is often a non-convex function .
- For acquisition functions with high exploitation , Bayesian Optimization often ends up getting stuck in local maximum .

Optima: $x=-1.310$, $y=4.039$



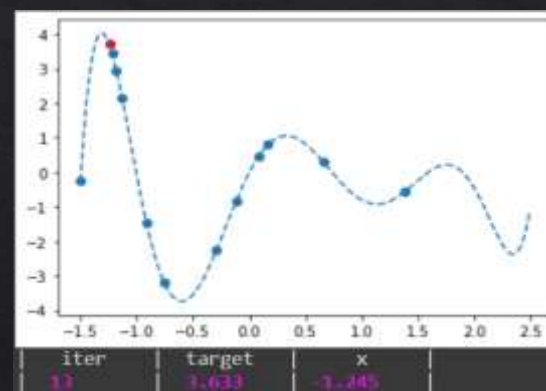
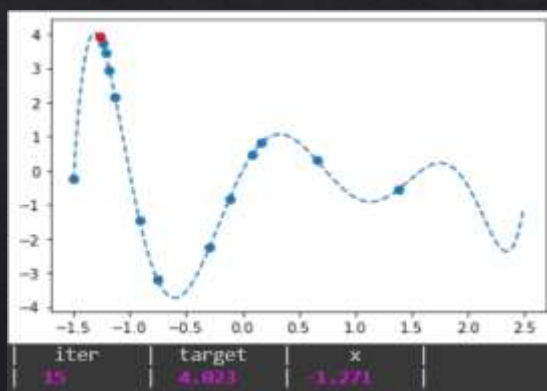
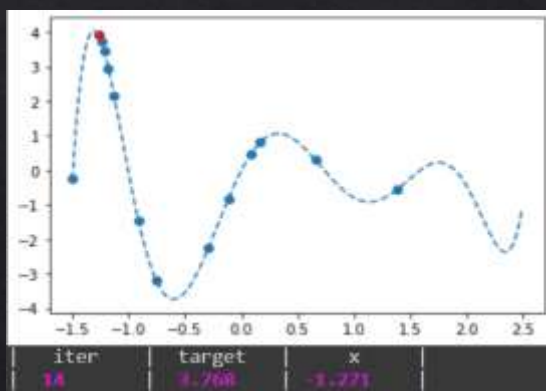
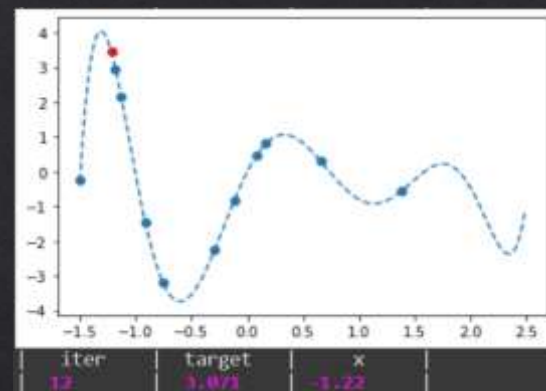
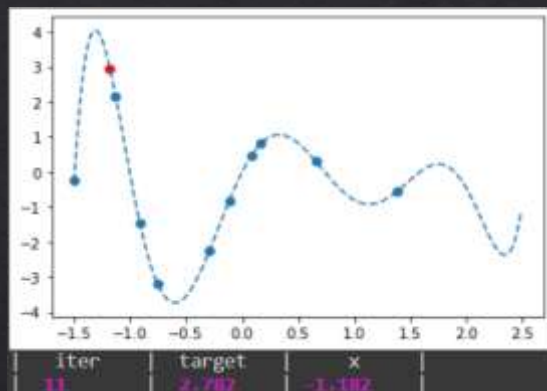
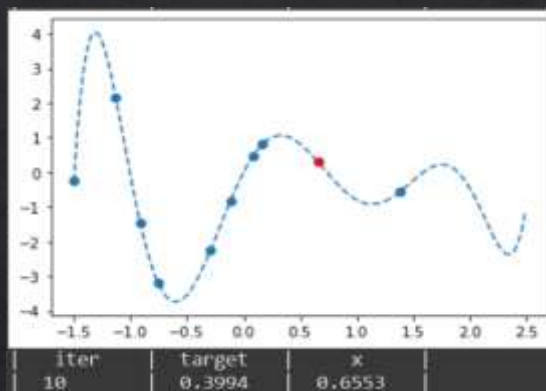
Problem of local maximum



Problem of local maximum

- **To strike a balance between exploitation and exploration:**
 - à Combine the strategy of grid search.
 - à Probe data points according to uniform distribution models .

Problem of local maximum





S u m m a r y

Tuning Hyperparameters with Bayesian Optimization



THANK YOU!

Tuning Hyperparameters with Bayesian Optimization